Albahit Journal of Applied Sciences
"open-access, peer-reviewed biannual journal"
eISSN: 2708-8936, pISSN: 2708-244X
Volume 5, Issue 3, 2026
Page No: 24-34
***Website**: https://albahitjas.com.ly/index.php/albahit/en/index*

Optimal Design of Double Tuned Filter Using LAGRANGE interpolation method to Attenuate THD in Power System

Mohamed.M.Almelian[1*], Elhadi Emhemed Aker[2], Mohammad A Omran[3], N. M. Elasager[4], Ibrahim Mohamed Yosser[5]
[1, 5] Department of Electrical Engineering, Higher Institute of Science and Technology Mezdah, Libya
[2] Department of Electrical Engineering, Higher Institute of Science and Technology Gharian, Libya
[3, 4] Faculty of Engineering, College of Technical Sciences, Ben-Walid, Libya
[*]*Corresponding author: almalyan84@gmail.com*

Received: 15-04-2026	**Accepted:** 02-07-2026	**Published:** 12-07-2026

Abstract:
Nonlinear loads, including power electronic converters and arc furnaces, are now widely deployed across both industrial and residential settings. As the penetration of such loads increases, the harmonic currents injected back into the supply also grow, raising the total harmonic distortion (THD) and causing a range of faults in electrical equipment. Among the available mitigation techniques for current harmonics is the double-tuned passive filter. This type of filter reduces reactive power by utilizing passive elements to supply local reactive power compensation at the fundamental frequency. This work presents an optimal design of a double-tuned filter using the Lagrange interpolation method to suppress harmonics within a power system. Lagrange interpolation is employed to obtain the filter's optimal parameters. Simulation carried out in MATLAB SIMULINK confirms that the resulting design fulfill to recognized limits such as those of IEEE 519.

Keywords: Double tuned passive filter, Total harmonic distortion, Nonlinear loads, Lagrange method.

التصميم الأمثل لمرشح مزدوج الضبط باستخدام طريقة استيفاء لاغرانج لتقليل التشوه التوافقي الكلي (*THD*) في نظام القدرة الكهربائية

الملخص
تنتشر **الأحمال غير الخطية**، بما في ذلك محولات القدرة الإلكترونية وأفران القوس الكهربائي، على نطاق واسع في كلٍّ من البيئات الصناعية والسكنية. ومع زيادة انتشار هذه الأحمال، تزداد **التيارات التوافقية** المحقونة مرة أخرى في شبكة التغذية، مما يؤدي إلى ارتفاع **التشوه التوافقي الكلي (THD)** والتسبب في مجموعة من الأعطال في المعدات الكهربائية.
ومن بين التقنيات المتاحة للحد من التوافقيات الحالية، يأتي **المرشح السلبي مزدوج الضبط (Double-Tuned Passive Filter).** حيث يعمل هذا النوع من المرشحات على تقليل القدرة غير الفعالة من خلال استخدام عناصر سلبية لتوفير تعويض محلي للقدرة غير الفعالة عند التردد الأساسي.
يقدم هذا العمل **تصميماً أمثل لمرشح مزدوج الضبط باستخدام طريقة استيفاء لاغرانج بهدف تخفيض التوافقيات في نظام القدرة الكهربائية**. حيث تم استخدام **استيفاء لاغرانج** للحصول على المعاملات المثلى للمرشح.
وقد أكدت المحاكاة التي أُجريت باستخدام برنامج **MATLAB SIMULINK** أن التصميم الناتج يحقق الحدود والمعايير المعترف بها، مثل تلك المحددة في المعيار **IEEE 519**.

الكلمات المفتاحية: المرشح السلبي مزدوج الضبط، التشوه التوافقي الكلي، الأحمال غير الخطية، طريقة لاغرانج.

Introduction

From a mathematical standpoint, a harmonic is defined through the ratio between its own frequency and that of the fundamental reference waveform [1]. The presence of harmonics gives rise to numerous problems in electrical apparatus, among them distortion of the voltage and current waveforms, excessive heating of motors, and erratic failures of computing equipment. Within industrial environments, harmonics are responsible for additional line losses, elevated transformer losses, and resonance phenomena, among other effects.

The principal origin of harmonic content in a network is the nonlinear load. Devices of non-linear load, such as fluorescent lighting, electric arc welders, and three-phase rectifiers, mainly produce the 5^{th} and 7^{th} current harmonics together with a number of higher-order components. Because harmonics in a power system cannot be avoided altogether, dedicated standards have to be drawn up to keep them within acceptable bounds so that the network operates as intended [2]. A number of guideline documents addressing harmonics have been issued by international organizations, notably IEEE 519 and IEC 61000 [3, 4].

Although harmonic distortion can never be removed completely, it may be lowered through a variety of approaches, including active filters and hybrid filters. Another widely adopted technique relies on the passive filter. Passive filters carry several merits: they are the simplest to implement, they require no external power supply, and they deliver the most favorable cost-to-benefit ratio of all mitigation options for low- and medium-voltage rectifier installations [5].

Representing an evolution of the single-tuned passive filter, the double-tuned filter is conceived to enhance the behavior of passive filtering, particularly that of the single tuned arrangement, while remaining more cost-effective [6].

In one study, a double-tuned harmonic passive filter was installed on low-voltage feeders and brought the current THD of the 3rd, 5th and 7th harmonics down to 0.90%, 0.10% and 1.67%, values that sit within the limits advised by IEEE 519 [7]. The work reported in [8] succeeded in lowering the voltage THD from 6.9% to 3.3%, the distortion having originated from the 7th and 11th harmonic orders introduced by a capacitor bank, while the power factor was corrected from 73.56% to 98.27% [8]. A novel damped double tuned filter aimed at improving power quality was proposed in [9], achieving a reduction of the current THD from 204% down to 3.57%. In [10], a hybrid passive arrangement that pairs a double tuned branch with a high-pass section was applied to harmonic mitigation in an industrial HVAC installation; the simulations reported there reduced the overall THD to 3.45% and the 3rd-order current THD to 3.42%, thereby satisfying IEEE 519 in full.

Despite the extensive literature on double-tuned passive filter design, a notable gap persists in achieving a single optimization that reconciles demanding harmonic limits, such as those of IEEE 519, with low filter costs. This challenge arises because driving the Total Harmonic Distortion (THD) below 5% typically requires larger, more expensive components. Therefore, this study proposes an optimal design for a double-tuned filter based on the Lagrange interpolation method. This approach strikes a balance between compliance with harmonic standards and cost-effectiveness for efficient harmonic suppression.

Passive Filter

The passive filter is the technique most frequently used to cancel harmonic currents within a distribution network. The passive harmonic filter is built on either single-tuned or band-pass filtering principles [11,12].

True to its description, the shunt filter is placed in parallel with the load. At the frequency to which it is tuned, the filter presents a very low impedance to the network, thereby diverting the corresponding harmonic current. Since a passive filter inherently supplies a degree of reactive power, a shunt design serves two objectives at once: it filters the targeted harmonics and, at the same time, provides reactive compensation to bring the power factor to the desired level [12]. The basic configuration of a passive filter in a power system is depicted in Figure 1, comprising the voltage source, the nonlinear load, and the passive filter itself. A sinusoidal supply (Va, Vb, Vc) feeds the nonlinear load through a transmission line characterized by three inductances (La, Lb, Lc). The nonlinear load distorts this sinusoidal waveform.

The compensation of the resulting distorted, harmonic-laden current is accomplished by a passive filter connected in parallel between the supply and the load. This filter is formed from a combination of resistive, inductive, and capacitive elements. Passive filters exist in several forms, the most common of which are illustrated in Figure 2 [13].

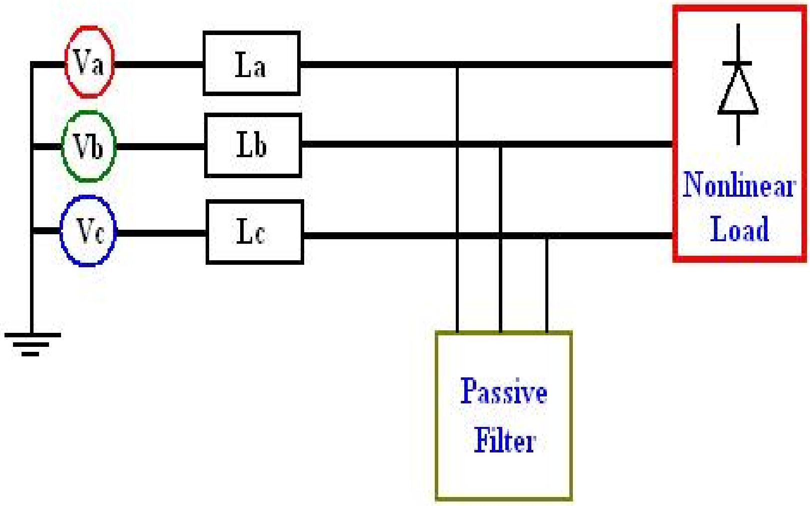

Figure 1. Basic scheme of passive filter

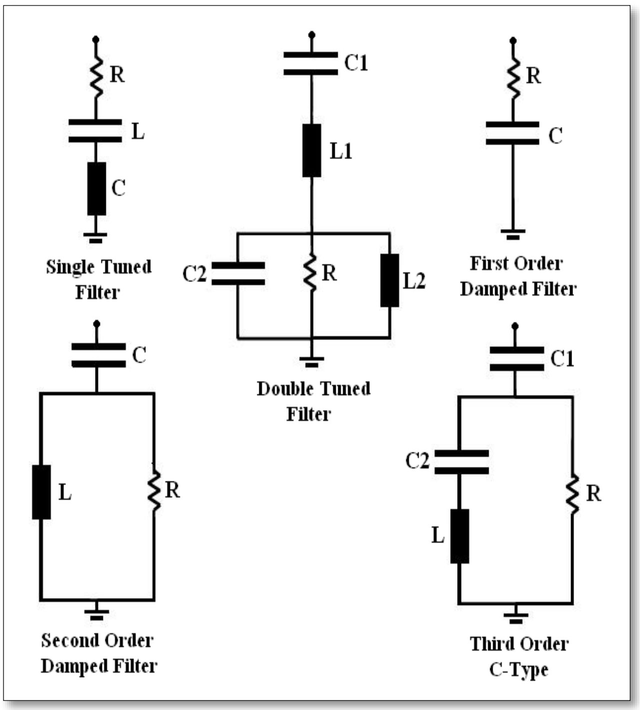

Figure 2. Different type of passive filters

Lagrange interpolation method

In many practical situations, the available data take the form of discrete points such as $(m_{0,},n_0)$, $(m_{1,},n_1)$, …., $(m_{k,},n_k)$. To describe them, one constructs a function spanning all $(k+1)$ points as a continuous curve $f(m)$ passing through the $(k+1)$ data points, as illustrated in Figure 3.

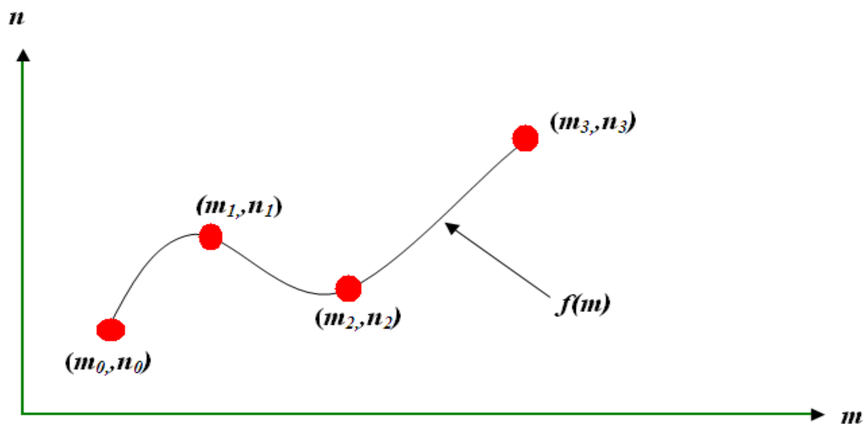

Figure 3. Interpolation of discrete data

Interpolation makes it possible to estimate the value of (n) corresponding to any intermediate value of (m). When $f(m)$ is evaluated beyond the span of (m) covered by the supplied data, the procedure is instead termed extrapolation. Polynomials are a popular choice of interpolating function, since, compared with alternatives such as trigonometric or exponential series, they are straightforward to evaluate, differentiate, and integrate. Polynomial interpolation amounts to determining a polynomial of order (k) that passes through the $(k+1)$ given data points, and one established technique for obtaining it is the Lagrange interpolation method. The Lagrange interpolating polynomial is expressed as [14]:

$$f_k(m) = \sum_{i=0}^{k} L_i(m) f(m_i) \tag{1}$$

Where (k) in $f(m)$ denotes the (k^{th})- order polynomial approximating the function $n = f(m)$ given at $(k+1)$ data points as $(m_{0,},n_0)$, $(m_{1,},n_1)$,…., $(m_{k,},n_k)$, and

$$L_i(m) = \prod_{\substack{j=0 \\ j\neq i}}^{k} \frac{m - m_j}{m_i - m_j} \tag{2}$$

Where $L_i\,(m)$ denotes a weighting term formed from the product of $(k\text{-}1)$ factors, the term for which $j=i$ omitted.

Optimal design of double tuned filter

A double-tuned filter is realized by combining series- and parallel-connected passive elements. With a single circuit, it is able to suppress two lower-order harmonics (the 3^{rd},5^{th}, 7^{th}, and so

on), whereas a single tuned design would need two independent parallel branches to achieve the same effect [15].
The double-tuned filter is placed in parallel with the distribution network, where it presents a low impedance path that draws the harmonic currents away from the system. A basic configuration of the double-tuned passive filter appears in Figure 4 [16].

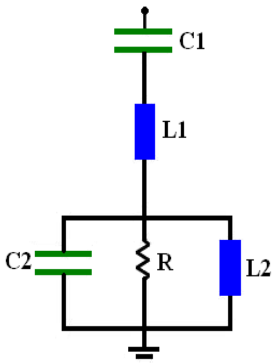

Figure 4. Simple arrangement of double-tuned filter

Designing the double tuned passive filter requires determining suitable values for a series branch (L1, C1) and a parallel branch (R, L2, C2) such that the harmonics present in the power system are attenuated.
Here, optimal design refers to keeping the filter cost to a minimum by selecting the smallest capacitor, inductor, and resistor values that still yield a THD of 5%, this being the maximum permitted by the IEEE 519 standard [10].

The procedure for designing the double tuned filter proceeds as follows:

1. Determine the three-phase capacitive reactive power (Qc) in VArs, taken as a positive quantity. To this end, Qc was swept from 1000 to 9000 VArs in increments of 1000, and the THD was recorded for every value of Qc, where the Qc bears an inverse relationship to the THD. The optimal value of (Qc) was then identified through Lagrange interpolation (equations (1) and (2)) as the one corresponding to a THD of 5%, noting that the filter size grows in direct proportion to Qc.

2. Obtain the capacitive reactance (Xc) together with the series capacitance (C) at the fundamental frequency (f).

$$X_C = \frac{V^2}{Q_c} \times \frac{n^2}{n^2 - 1} \tag{3}$$

$$\mathrm{C}_1 = \frac{1}{2\pi f X_C} \tag{4}$$

Where: V= The (rms) nominal line to line voltage (in V).
n= The harmonic order desired.

3. Compute the inductive reactance (XL) along with the series inductance (L1) at the fundamental frequency.

$$X_L = \frac{X_C}{n^2} \quad (5)$$

$$L_1 = \frac{X_L}{2\pi f} \quad (6)$$

4. The series branch (C1, L1) is computed for the first desired nth harmonic order, after which the parallel branch (C2, L2) is obtained for the second desired nth harmonic order of the double tuned filter, applying the same relations given above.

5. Determine the resistance (R) of the parallel branch from a chosen quality factor (Q) and the geometric mean frequency (fm), with Q lying in the range of 30 to 50.

$$\mathrm{R} = Q \times L_2 \times 2\pi f_m \quad (7)$$

$$f_m = \sqrt{f1f2} \quad (8)$$

where: f1= The first tuning frequency.
f2= The second tuning frequency.

Parameters of power system model with double-tuned filter design

The circuit parameters of the power distribution network illustrated in Figure 5 are listed in Table 1 [12], where the variables of the power circuit have been chosen to create a harmonics issue. The optimal values of the double tuned filter (C1, L1, C2, L2, R) are presented in Table 2; these parameters of the double tuned passive filter were obtained from equations (3) through (8) at Q=30 with a 5% THD target in line with IEEE Standard 519. Tuning was set to the 5^{th} and 7^{th} harmonics, since these exhibited the largest magnitudes relative to the fundamental component.

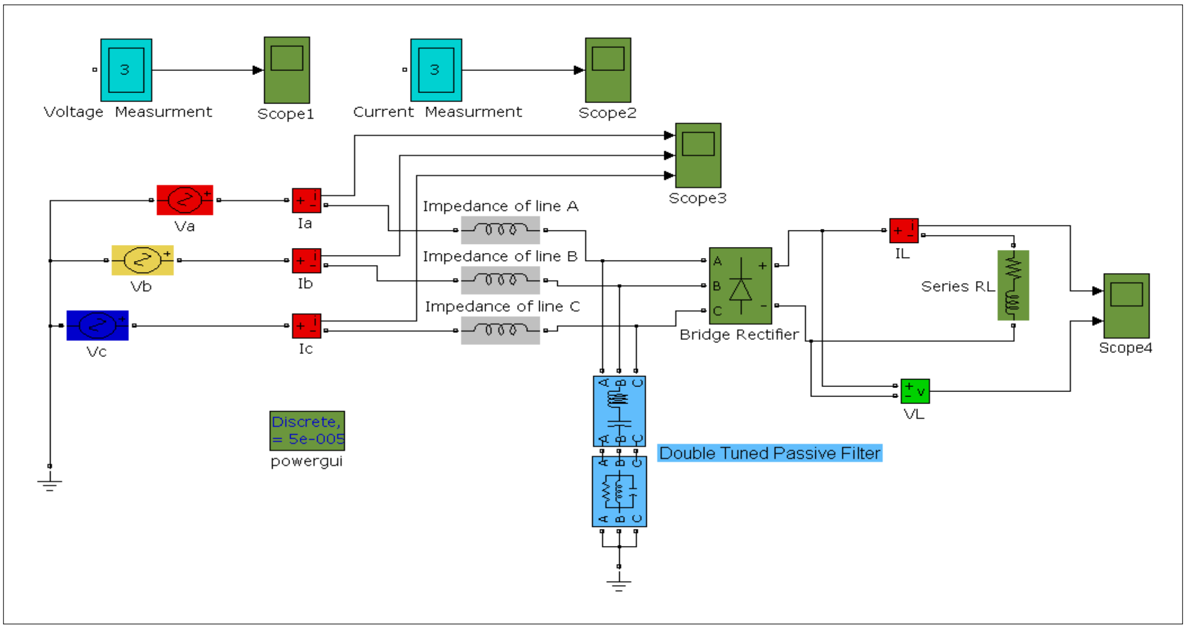

Figure 5. Power system model with double-tuned filter

Table 1 Test circuit parameters of power system

Parameter	Values
Network voltage	V = 220 V
Network frequency	f = 50 Hz
Line inductance	Labc = 1.7 mH
Rectifier load inductance	LL = 1 mH
Rectifier load resistance	RL = 100 Ω

Table 2 Optimal values of double-tuned passive filter

C1 (mF)	*L1 (mH)*	*C2 (mF)*	*L2 (mH)*	*R (Ω)*
0.4545	0.8916	0.4638	0.4458	24.8

Results analysis and discussion

The effectiveness of the chosen double-tuned passive filter parameters was verified in MATLAB. A nonlinear load represented by a three-phase bridge rectifier was modeled both with and without the double-tuned passive filter present. The corresponding MATLAB simulation circuit, noted earlier, appears in Figure 5.

The supply voltage and the line current waveforms in the absence of the double tuned filter are presented in Figure 6 and Figure 7, respectively. Figure 7 reveals that the line current is visibly nonlinear. A fast Fourier transform (FFT) was therefore applied to extract its frequency spectrum and to quantify the THD, the result of which is given in Figure 8; the THD is found to be approximately 28.28%.

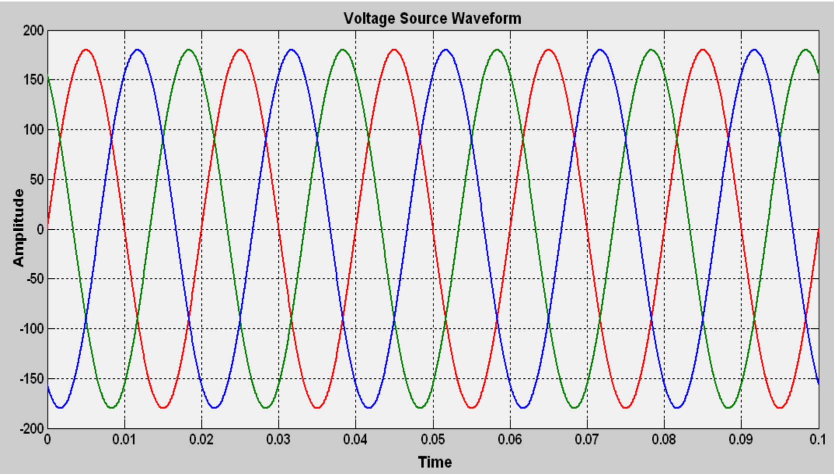

Figure 6. Voltage source without double-tuned filter

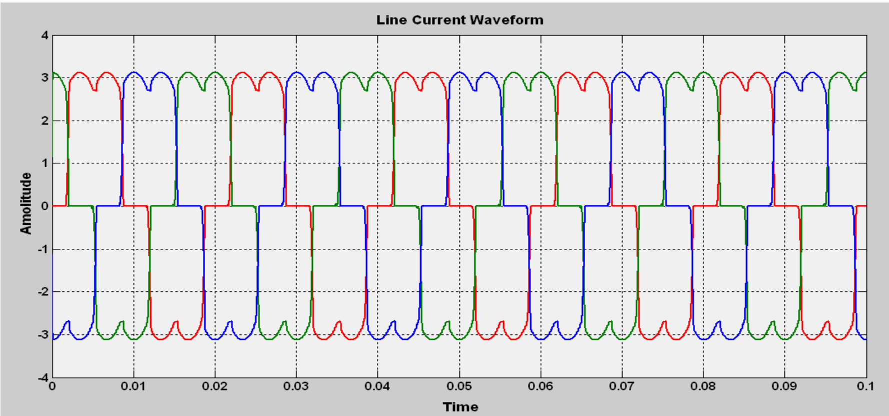

Figure 7. Line current without double-tuned filter

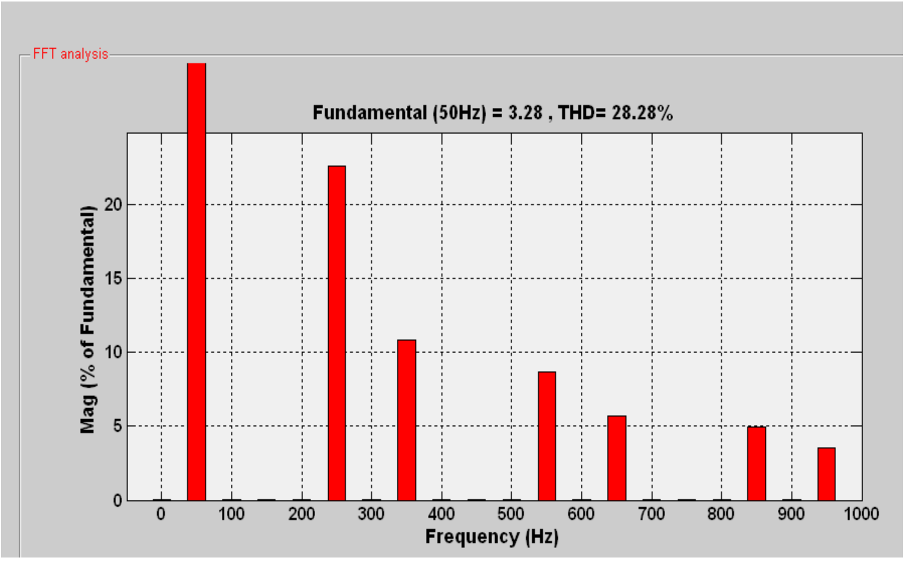

Figure 8. Frequency spectrum of the line current without double-tuned filter

The harmonic magnitudes, expressed as percentages of the fundamental component with the optimal double-tuned filter disconnected, are set out in Table 3.

Table 3. Harmonic values in the line current with respect to the fundamental component (without double-tuned filter)

Fundamental (50Hz) = 3.28 A THD = 28.28%			
Harmonic order	**% of fundamental harmonic**	**Harmonic order**	**% of fundamental harmonic**
1	100	11	8.64
2	0.03	12	0.03
3	0.05	13	5.69
4	0.03	14	0.03
5	22.59	15	0.05
6	0.03	16	0.02
7	10.83	17	4.94
8	0.03	18	0.02
9	0.05	19	3.52
10	0.03	20	0.00

Figure 9 presents the line current waveform with the double-tuned passive filter in service. The waveform is now essentially a pure sinusoid, confirming that the filter has suppressed the harmonics.

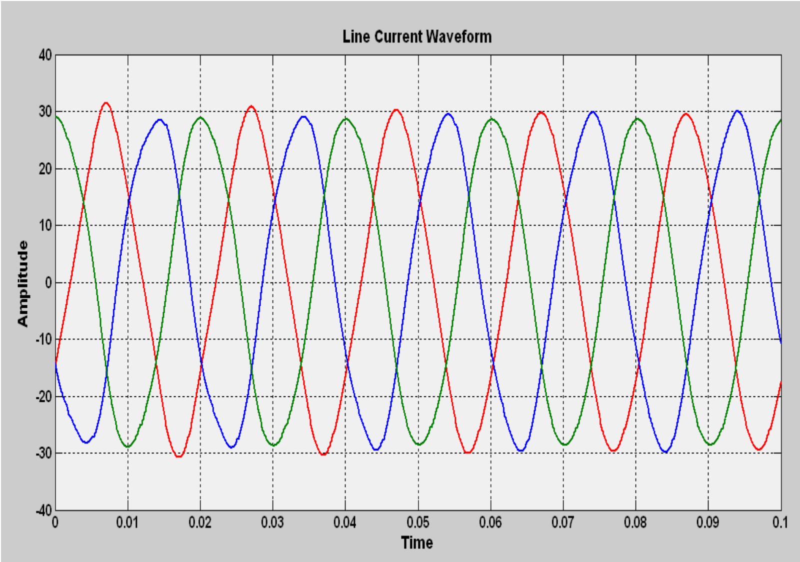

Figure 9. Line current with double-tuned filter

As shown by the line current frequency spectrum in Figure 10, the 5th, 7th harmonics have been removed, while the 3rd has grown relative to Figure 8.

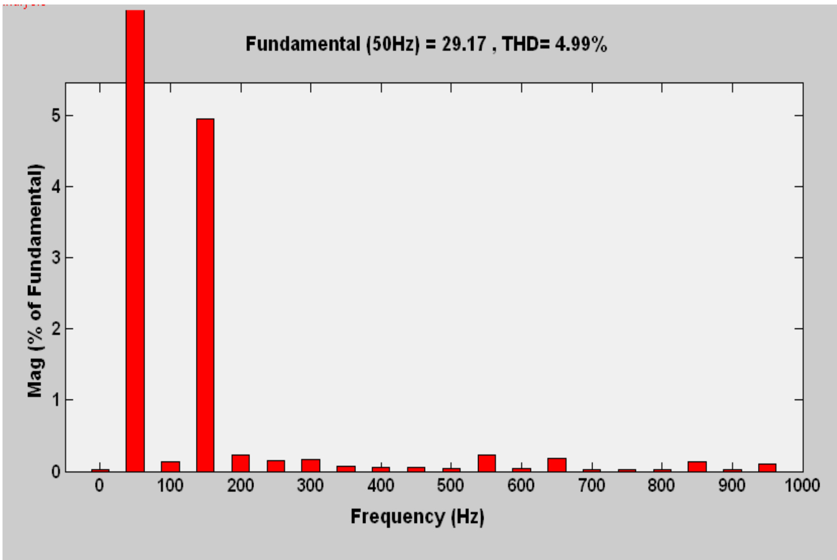

Figure 10. Frequency spectrum of the line current with double-tuned filter.

Table 4 reports the harmonic content, in percent of the fundamental, once the filter is connected. The 5th and 7th harmonics fall from 22.59% to 0.15% and from 10.83% to 0.07%, respectively, so that the overall THD drops from 28.28% to 4.99% once the double tuned passive filter is introduced. Here, the 3rd harmonic was increased from 0.05% to 4.96% due to unsuitable impedance shifting at lower frequencies. In the context of harmful harmonic orders, the 3rd harmonic in [11] was also grown to 4.79%. The line current amplitude, on the other hand, rises from 3.28 A to 29.17 A as a consequence of the filter impedance; for comparison, the authors of [16] employed four single tuned filters and observed the source current rising from 6.35 A to 13.06 A.

Table 4. Harmonic values in the line current with respect to the fundamental component (with double-tuned filter)

Fundamental (50Hz) = 29.17 A THD = 4.99%			
Harmonic order	**% of fundamental harmonic**	**Harmonic order**	**% of fundamental harmonic**
1	100	11	0.24
2	0.14	12	0.03
3	4.96	13	0.18
4	0.23	14	0.03
5	0.15	15	0.02
6	0.17	16	0.03
7	0.07	17	0.14
8	0.06	18	0.02
9	0.05	19	0.11
10	0.04	20	0.00

Conclusion

This work has set out an optimal design of a double tuned filter, based on Lagrange interpolation, for suppressing the harmonic distortion produced by nonlinear loads in a power distribution network. The simulation outcomes indicate that the double tuned passive filter design successfully reconciled compliance with harmonic standards and overall cost. In particular, the line current THD was lowered from 28.28% to 4.99%, and the design was carried out so as to keep the dimensions of all the passive components (inductors, capacitors, and resistor) as small as possible. A THD of 4.99% lies within the recommended IEEE 519 limit. The line current amplitude, by contrast, increased from 3.28 A to 29.17 A owing to the impedance of the double tuned passive filter. Examining the harmonic levels before and after the filter was applied, the 5th harmonic fell from 22.59% to 0.15%, the 7th from 10.83% to 0.07%, and the 11th from 8.64% to 0.24%. In summary, the effectiveness of the double tuned passive filter in attenuating harmonics rests on the chosen values of its parameters, namely C, L, and R in suppressing the harmonics.

References

1. Grady, W. Mack, and Surya Santoso. "Understanding power system hannonics." IEEE Power Engineering Review 21.11 (2001): 8-11.
2. Yong, B. H., and V. K. Ramachandaramurthy. "Double Tuned filter design for harmonic mitigation in grid connected solar PV." Power and Energy (PECon), 2014 IEEE International Conference on. IEEE, 2014.
3. IEEE Standard Association. "519-1992. IEEE Recommended Practices and Requirements for Harmonics Control in Electric Power Systems." New York: IEEE Inc (1992).
4. McGranaghan, M., and G. Beaulieu. "Update on IEC 61000-3-6: Harmonic emission limits for customers connected to MV HV and EHV." Transmission and Distribution Conference and Exhibition. 2006.
5. Fujita, Hideaki, Takahiro Yamasaki, and Hirofumi Akagi. "A hybrid active filter for damping of harmonic resonance in industrial power systems." IEEJ Transactions on Industry Applications 118.10 (1998): 1193-1200.

6. HE, Yi-hong, and Heng SU. "A New Method of Designing Double–tuned Filter." Proc. of the 2nd International Conference on Computer Science and Electronics Engineering (ICCSEE 2013). 2013.
7. Tan, Jun Zhe, and Rodney HG Tan. "Double-tuned harmonic passive filter design for harmonic mitigation at LV distribution network caused by electric vehicle on-board charger." 2023 Innovations in Power and Advanced Computing Technologies (i-PACT). IEEE, 2023.
8. Widagdo, Reza Sarwo, Kukuh Setyadjit, and Izzah Aula Wardah. "Analysis and Mitigation of Harmonics Distortion with Optimization Capacitor Banks and Single-Tuned Passive Filters." Jambura Journal of Electrical and Electronics Engineering 5.2 (2023): 204-209.
9. Pasaribu, Faisal Irsan, et al. "A New Damped Double-Tuned Filter to Improve Power Quality and System Performance for Nonlinear Household Loads." IEEE Open Access Journal of Power and Energy 13 (2026): 76-87.
10. Malik, Mochamad Irlan, et al. "Hybrid Double-Tuned and High-Pass Filter for Harmonic Mitigation in Inverter HVAC Systems." ELECTRON Jurnal Ilmiah Teknik Elektro 6.2 (2025): 139-152.
11. Soomro, Dur Muhammad, and M_ M_ Almelian. "Optimal design of a single tuned passive filter to mitigate harmonics in power frequency." (2015).
12. Memon, Zubair Ahmed, Mohammad Aslam Uquaili, and Mukhtiar Ali Unar. "Harmonics mitigation of industrial power system using passive filters." arXiv preprint arXiv:1605.06684 (2016).
13. Berrut, Jean-Paul, and Lloyd N. Trefethen. "Barycentric lagrange interpolation." Siam Review 46.3 (2004): 501-517.
14. Cheepati, Kumar Reddy, Sardar Ali, and M. Surya Kalavathi. "Performance Analysis of Double Tuned Passive Filter for Power Quality." International Journal of Control Theory and Applications9.07 (2016): 3295-3305.
15. Dastfan, Ali, Hossain Yassami, and Mohammad Reza Rafiei. "Optimum design of passive harmonic filter by using game theory concepts." Intelligence Systems in Electrical Engineering Journal4.4 (2014): 13-22.
16. Anooja, C. L., and N. Leena. "Passive Filter For Harmonic Mitigation Of Power Diode Rectifier And SCR Rectifier Fed Loads."International Journal of Scientific & Engineering Research 4.6 (2013).
17. Modeling and Simulation of Energization Overvoltage Phenomena During Capacitor Switching Transient. (2026). *Albahit Journal of Applied Sciences*, *5*(1), 126-140. https://doi.org/10.65419/albahit.v5i1.122
18. Modeling the Performance and Frictional Head Losses of the Control Oil Pump at Derna Steam Power Plant. (2025). *Albahit Journal of Applied Sciences*, *4*(1), 161-170. https://doi.org/10.65419/albahit.v4i1.54
19. Quantitative Analysis of Renewable Energy Systems Efficiency and Reliability: A Meta-Study of Solar, Wind, and Energy Storage Experimental Data. (2025). *Albahit Journal of Applied Sciences*, *4*(1), 9-17. https://doi.org/10.65419/albahit.v4i1.38

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.