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# Optimization of Modes in Variational Mode Decomposition Based on Correlation Coefficients

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***Abstract*—Enhancement of signal is highly essential to either remove or reduce the noise from it. More often we meet these difficulties due to nonlinearities in signal and due to its nonstationary nature. Fourier transform is a popular and efficient tool for this purpose along with wavelet transform. It may not work properly to its limitations, for the nonstationary type of signal it fails to prove its efficiency. Also it cannot offer the temporal and spatial information clearly. The empirical mode decomposition EMD of signal is an adaptive method and a powerful substitution to the Fourier and wavelet transform. This technique can be used for an effective way of analysis of the instantaneous frequency of signals. Though EMD has used by many researchers, it cannot be most popular due to its demerits in terms accurate mathematical model. Therefore the birth of variational mode decomposition VMD occurred as an alternative of EMD and can overcome the demerits of EMD. VMD decomposes the signal into discrete number of sub-signals (modes), where each mode has limited bandwidth in spectral domain. But the problem of this technique is how we can define the optimal number of modes where too large number of modes will lead to redundant VMD information, while too small number of modes will lead to mode mixing in the VMD results. In this paper we propose a combination of VMD and a correlation coefficients (CCs) to optimize the number of modes based on estimation of useful modes to reconstruct the original signal and determination of noisy modes to be removed. The robustness of VMD is evaluated on simulated signals under different parameters and the performance of the method for signal denoising is evaluated in terms of signal-to- noise ratio (SNR).**

***Index Terms*—Variational mode decomposition, Empirical mode decomposition, correlation coefficients.**

## I. Introduction

Analysis of signal is a vital part for industry, research and academia. It is the great challenge for researchers in current decade. Different types of analysis can be performed based on users' requirement. Signal estimation as well as estimation of its parameter is able to identify and confirm the model to be used, where noise is mostly contaminated with it. For enhancement of signal, it is very essential to either remove or reduce the noise from it. More often we encounter these difficulties due to nonlinearities in signal and due to its nonstationary nature. For such purpose spectral analysis is an alternative. But accuracy level has to be taken care. Fourier transform is a popular and efficient tool for this purpose along with wavelet transform. Sometimes these methods are effective for specific cares and specific signals. Consideration of Fourier transform may not work properly to its limitations, for the nonstationary type of signal it fails to prove its efficiency. Also it cannot offer the temporal and spatial information clearly. Though it can be solved using wavelet transform, still it has certain demerits. The empirical mode decomposition EMD of signal is an adaptive method and a powerful substitution to the Fourier and wavelet transform. EMD technique has been suggested for nonlinear and nonstationary signals [1]. After decomposition of the signals, it can be reconstructed as the sum of the components along with the amplitude and frequencies parameters. It can be said that the multi resolution method to perform space-spatial frequency decomposition as time-frequency analysis. This technique can be used for an effective way of analysis of the instantaneous frequency of signals. Though EMD has used by many researchers, it cannot be most popular due to its demerits in terms accurate mathematical model. Also choice of interpolation, sensitivity to noise and sampling are the factors of demerits. Therefore the birth of variational mode decomposition VMD occurred as an alternative of EMD and can overcome the demerits of EMD [2-4]. Its basic principle is same as EMD, but the centre frequency of the mode has to be found so that the bandlimited modes can represent the original signal. VMD used for different applications like classification and detection [5-7]. Variational Mode Decomposition VMD decomposes the signal into Z discrete number of subsignals (modes), where each mode has limited bandwidth in spectral domain. Thus, each mode Z is required to be mostly compact around a centre pulsation $\omega z$ determined along with the decomposition. One of the disadvantages of VMD is how to determine the number of intrinsic mode functions IMFs where the number of modes Z must be predefined before starting the VMD calculation. Therefore, it is very necessary to optimize the Z value to ensure the accuracy and efficient computation of the VMD. In this paper we propose a combination of VMD and correlation coefficients (CCs) to define an appropriate value of Z. By this combination we can define the necessary modes IMFs to reconstruct the original signal and define the noisy modes IMFs to be removed.

## II. OPTIMIZATION OF THE NUMBER OF MODES

The number of modes Z must be predefined prior the VMD computation starts. Theoretically, the value of Z depends on how many components are present in the signal. When the value of Z is improperly realized, the unneeded computations are increased, and at the same time the accuracy of signal separation may be affected [2]. Redundancy VMD information is caused when too many modes are selected and mode mixing results in VMD will appear for too few modes. Therefore, it is very important to find the optimal value of Z to get high accuracy and efficient computation of the VMD. The question of how to find the optimal value of Z has not yet been answered, but it remains to solve. The determination of Z depends on the experience of the researchers. In this paper, we assume that the number of mode functions (Z) is equal to the number of modes (IMFs) generated by empirical mode decomposition [1], followed by the decomposition method (VMD) jointed with correlated coefficients (CCs). Firstly, the simulation signal is decomposed into discrete number of mode functions equal to the same number of mode functions that decomposed by EMD method; the correlation coefficients between the input simulation signal and the generated intrinsic mode functions are computed and based on selected threshold value, we determine the useful CCs and discard the noisy mode functions (IMFs).

## III. CORRELATION COEFFICIENTS

The relationship between the two variables can be measured by correlation coefficients (CCs). The CCs are values ranging from (-1) to (1). If the relationship between the variables is completely positive linear, CCs is 1, but if the relationship is negatively related, CCs is -1. The correlation coefficient is zero when the relationship between the variables is not linear. There are two kinds of correlation coefficients; one of them is Spearman's rank correlation coefficient, which is depends on the rank association between the amounts. The second technique is the most popular technique for measuring the relationship between two quantities which is called Pearson's correlation coefficient. Pearson's correlation coefficient (r), sometimes referred to as the correlation coefficient or r, is the most popular technique used to optimize the relationship between two quantities in which this relationship is positively correlated, negatively correlated or nonlinear relationship (not a straight line). The (r) values of the two elements are frequently reported in research articles and journals, to summarize the relationship between two elements. The value (r) will be positive and much greater than zero, when the two quantities have a linear straight line relationship in the positive direction. The value (r) is less than zero, when the relationship is straight line in negative. When the values of (r) are very close to zero then the relationship between the two quantities is small. The range of values (r) is from -1 to 1. To calculate Pearson's correlation coefficient (r), assumed that we have two quantities u and v.

$$r = \frac{SS_{uv}}{\sqrt{(SS_{uu})(SS_{vv})}} = \frac{\sum_{i=1}^{N}(u_i-\bar{u})(v_i-\bar{v})}{\sqrt{\sum_{i=1}^{N}(u_i-\bar{u})^2 \sum_{i=1}^{N}(v_i-\bar{v})^2}} \quad (1)$$

In order to find the correlation coefficient of two quantities, we usually need three additions. The sum squares of the quantity (u), sum squares of the quantity (v) and the cross product sum of (uv)

Let the mean of u be

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i \quad (2)$$

And the mean of v

$$\bar{v} = \frac{1}{N}\sum_{i=1}^{N} v_i \quad (3)$$

The sum squares of quantity (u) is

$$SS_{uu} = \sum_{i=1}^{N}(u_i - \bar{u})^2 \quad (4)$$

The sum squares of quantity (v) is

$$SS_{vv} = \sum_{i=1}^{N}(v_i - \bar{v})^2 \quad (5)$$

The cross-products (*SS*uv) sum

$$SS_{uv} = \sum_{i=1}^{N}(u_i - \bar{u})\,(v_i - \bar{v}) \quad (6)$$

The relationship strength between the variables is estimated by the magnitude of the correlation coefficient:

$0 < |r| < .3$ weak correlation

$.3 < |r| < .7$ moderate correlation

$|r| > 0.7$ strong correlation

## IV. SIMULATIONS AND RESULTS

The simulation signal *x(t)* is composed of three different frequency and amplitude cosine signals. A 0.5 times standard Gaussian white noise *n(t)* is added to get the noisy signal *f(t)*. The simulation signals are as follows

$x(t) = 0.8\cos(2\pi f1t) + 0.6\cos(2\pi f2t) + 0.3\cos(2\pi f3t)$ (7)

$n(t) = 0.5\text{rand}\, n(t)$ (8)

$f(t) = x(t) + n(t)$ (9)

where f1 = 10, f2 = 50 and f3 = 100 represent the three frequencies of clear signal *x(t)*; and *f(t)* is the noisy signal containing both *x(t)* and *n(t)*. The time-domain waveform for clear signal and noisy signal is shown in Figure 1.

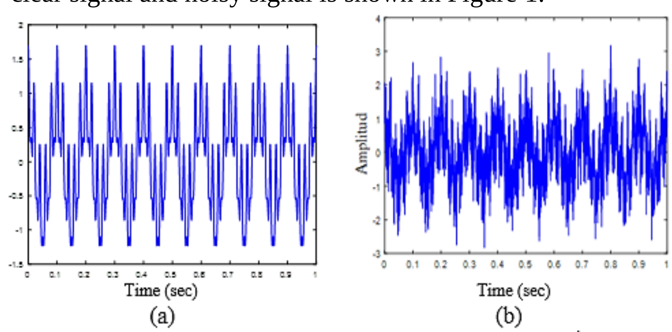


Fig. 1. (a) The clear signal; (b) The noisy signal.

VMD is used to remove or reduce the noise from the noisy signal corrupted with WGN. The noisy signal is first broken down into the number of modes Z=9. The number of modes in VMD is selected to be equal to the number of modes by empirical mode decomposition (EMD) as a default [1]. This number of modes Z=9 is used on VMD as an initial value of Z. The decomposition modes of VMD for a noisy signal are presented in Figure 2. Optimization of these modes is needed to avoid the impact of over-binning or under-binning on the VMD denoising method. A simple criterion is designed based on a combination of VMD and correlation coefficient (CCs), to minimize the number of modes Z. VMD algorithm is presented to decompose the noisy signal into a nine band limited modes then, a CCs rule is designed to identify the effective components from these modes.

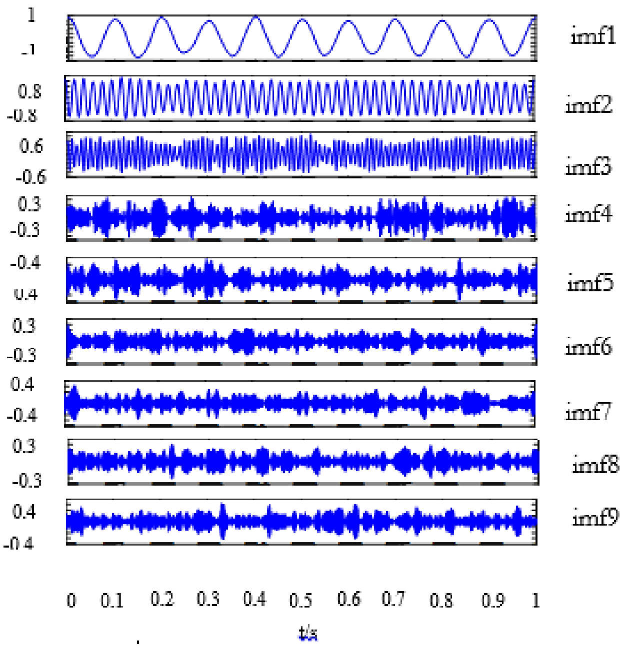


Fig. 2. The IMFs of noisy signal.

After simulation signal is decomposed into intrinsic mode functions (IMFs), then the IMFs are defined as a noise mode functions and useful mode functions, respectively. The distinction between noise mode functions (IMFs) and useful mode functions (IMFs) can be made according to the threshold of the (CCs). The formula of correlation coefficient (CCs), as statistical parameter is shown as following:

$$r = \frac{\sum(f-\bar{f})(u_z-\bar{u}_z)}{\sqrt{\sum(f-\bar{f})^2\sum(u_z-\bar{u}_z)^2}} \tag{10}$$

Where $f$ is the noisy simulation signal and $u_z$ is mode components of VMD. The objective of this simulation is to optimize the number of decomposition modes Z, by CCs and measure the performance of different input SNRs. The CCs are computed for each mode of the decomposition modes (in Figure 2) with respect to the noisy signal and the CCs results are recorded in Table 1. According to the selected threshold value (0.24), only three modes (IMF1, IMF2 and IMF3) were strong enough to be defined as useful modes and the other 6 modes (IMF4 – IMF9) were very weak which classified as noisy modes. The three effective modes are added together to get the reconstructed signal with SNR =15dB instead of 3dB for the noisy signal. The remains 6 modes are discarded as noisy components. The denoised signal is shown in Figure 3.

To measure the performance of variational mode decomposition (VMD) denoising algorithm, simulation experiments have been carried out in this paper, with different input SNRs in the range from −10 dB to 5 dB. The number of IMFs on VMD is set to Z=3, which is equal to the number of modes IMFs that optimized by CCs and the band width control parameter of VMD is $\alpha = 7000$. As seen in Figure 3, the reconstructed signal of VMD is improved after we make denoising, where the input SNRi =3 dB and the resultant SNRo=15.6 dB. The correlation coefficients (CCs) between the mode functions (IMFs) and the input noisy signal are recorded in Table 1, according to selected threshold which is 0.24. We get three useful IMFs by VMD (IMF1, IMF2, and IMF3).

TABLE I. THE CCS BETWEEN THE NOISY SIGNAL AND IMFS BY VMD

| IMF1 | IMF2 | IMF3 | IMF4 | IMF5 | IMF6 | IMF7 | IMF8 |
|---|---|---|---|---|---|---|---|
| **0.62** | **0.48** | **0.25** | 0.152 | 0.147 | 0.156 | 0.165 | 0.175 |

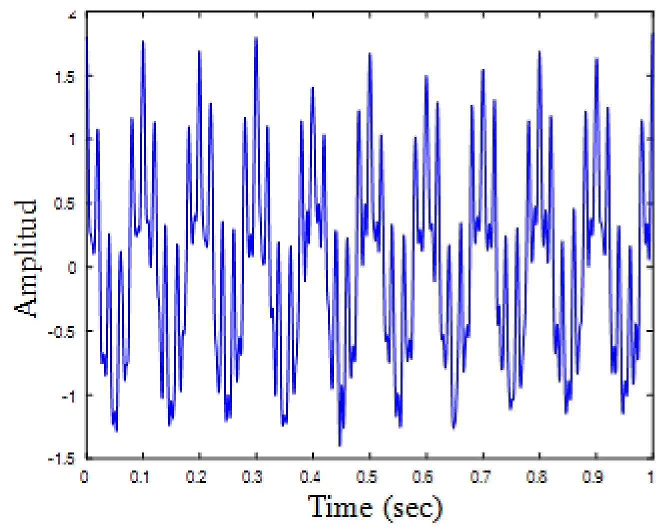


Fig. 3. Time-domain denoising signal (15.6dB).

The relationship between the two variables can be measured by correlation coefficients (CCs). The CCs are values ranging from (-1) to (1). If the relationship between the variables is completely positive linear, CCs is 1, but if the relationship is negatively related, CCs is -1.

To measure the achievement of VMD with respect to various input (SNRs) ranging from -10 dB to 5 dB, the variance (σ) is changed into different values (0.4 - 2.4). The output SNRs and RMSE results are recorded in Table 2. Figure 4 illustrates the plots of input SNRs versus output SNRs for simulation signal. The relation between input SNRi and output SNRo is proportional, when the input SNRi increases the output SNRo increases too and RMSE decreases.

TABLE II. DENOISING RESULTS OF NOISY SIGNAL WITH DIFFERENT SNRS

| $\sigma$ | 2.4 | 1.7 | 1.3 | 1.1 | 0.85 | 0.65 | 0.5 | 0.4 |
|---|---|---|---|---|---|---|---|---|
| SNRi | -10.4 | -7. 3 | -4.9 | -3.1 | -1.1 | 1.17 | 2.8 | 5.2 |
| SNRo | -0.68 | 2.62 | 5.2 | 7.7 | 10.7 | 13.1 | 14.6 | 18 |
| RMSE | 0.80 | 0.51 | 0.37 | 0.31 | 0.20 | 0.15 | 0.12 | 0.1 |

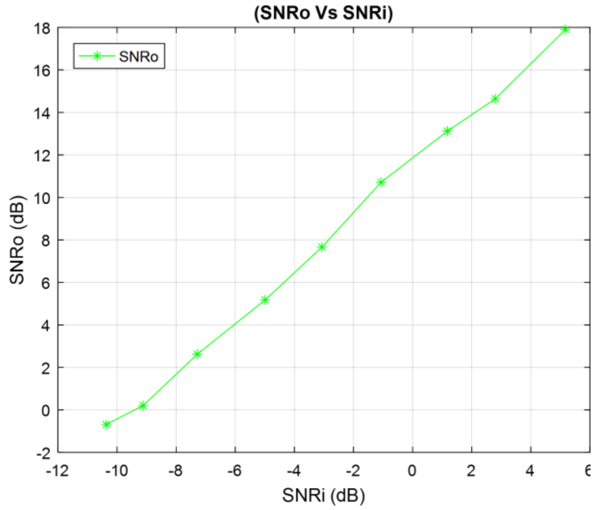

Fig. 4. Denoising results of noisy signal with different SNRs.

## V. CONCLUSION

VMD decomposes the signal into discrete number of sub-signals (modes), where each mode has limited bandwidth in spectral domain. But the problem of this technique is how we can define the optimal number of modes. For under-segmentation (small number of modes Z), there will be sharing between the neighbouring modes for small band width control parameter ($\alpha$), or mostly discarded modes for big band width control parameter ($\alpha$ ). On the other hand, when over-segmentation (large Z), and small band width control parameter ($\alpha$), one or several modes get noisy and broad spectral density. For large value of ($\alpha$ ), the center frequency of two or more modes get matched (mode duplication) and some of important parts of signal spectrum shared with others. In this paper we propose a combination of VMD and a correlation coefficients to minimize the number of modes (Z). We used a combination of VMD and correlation coefficient (CCs) to decompose the simulation signals into intrinsic mode functions (IMFs) and identify noise IMFs, respectively; According to the threshold of the CCs, noise IMFs and useful IMFs can be distinguished effectively. Then, the denoising can be realized by reconstructing useful IMFs. The performance of a VMD approach is measured over the simulation signal at different noise conditions ranging from -10 dB to 5 dB and the relationship between the input signal SNRs and the output signal SNRs is proportional, if the input SNR increases the output SNR increases and the RMSE values decreases.

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