Adaptive Digital FIR Filters: A Study; Case Study: Noise Cancellation using LMS Algorithm

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***Abstract* — In this paper, a review has been taken to the previous and the most recent studies and investigations on adaptive digital filter algorithms which based on adaptive noise cancellation systems. In numerous applications of noise cancellation such modern and adaptive systems characteristics that could be very quick in changing the situations which needs the utilization of adaptive filters that quickly changes accordingly. Algorithms such as LMS and RLS proves to be crucial within the noise cancellation are looked into counting guideline and later alterations to extend the merging rate and reduce the computational complexity for future execution. This paper, isn't only as a review of the basic principles on which of the adaptive filters are based uses least mean square LMS algorithm derivation of the least-mean-square (LMS) algorithm; but moreover; It's to implement a case-study of the adaptive filters to solve real-world application problems such as adaptive noise cancellation by implementing the LMS finite impulse response (FIR) adaptive filter using MATLAB, then, investigate of how to choose an appropriate value of convergence factor in order to achieve an efficient LMS adaptive filter.**

Keywords — Adaptive Filter, IIR, FIR, LMS, Noise Cancellation

I. INTRODUCTION

Due to stunning growth of the digital component technologies and demands all fields during the last years, the field of digital signal processing, and particularly adaptive signal processing, has developed hugely due to the increasingly availability of technology for the implementation of the emerging algorithms. These algorithms have been applied to an extensive number of problems including noise and echo canceling, channel equalization, signal prediction, adaptive arrays as well as many others.

"An adaptive filter may be understood as a self-modifying digital filter that adjusts its coefficients in order to minimize an error function. This error function, also referred to as the cost function, is a distance measurement between the reference or desired signal and the output of the adaptive filter" [1].Recently, the notion of noise cancellation has gained a lot of attention and has been recognized as an important method for removing/eliminating noise found in useful signals. This technique can be extended to a number of industrial and communication devices, such as machinery, hands-free telephones and transformers [2-4].

In addition, in the area of image processing, biomedical signal, voice enhancement and echo cancellation, noise cancellation was also introduced [5, 6]. As the noise from the ambient atmosphere significantly decreases the quality of speech and audio signals, it is very important to suppress noise and increase the quality of speech and audio signals, so the acoustic applications of noise cancellation have become the research focus field. The basic principle of the Adaptive Noise Canceller (ANC) was first introduced by Widrow [8], which eliminates or suppresses noise from a signal using adaptive filters. In the last few decades various methods have been suggested. For example, the Kalman filter [8] and the Wiener filter [9]; Recursive-Least-Square (RLS) [10] algorithms were proposed to achieve the optimum performance of adaptive filters. In this a paper a detailed study of the basic principles on which this adaptive digital finite impulse response FIR filter design is based, in turn to make it imposed in modern digital systems.

II. OVERVIEW ON DIGITAL FILTERS

Digital filters are used in wide variety of applications from signal processing, aerospace, control systems, defense equipment, telecommunications, system for audio and video processing to systems for medical applications to name just a few. Basically filter refers to a frequency selective device which extracts the useful portion of input signal lying within its operating frequency range and could be contaminated with random noise due to unavoidable circumstances. Analog filters are implemented with discrete components but the digital filters perform mathematical operations on a sampled, discrete time signal to reduce or enhance the desired features of the applied signal [11].

Digital filters are capable of performance specifications such as ability to achieve multi-rate operation and exact linear phase that would, at best, be extremely difficult, if not impossible, to achieve with an analogue implementation. Digital filters are classified into two types; finite impulse response (FIR) and infinite impulse response (IIR) filter which may also take on the names non-recursive and recursive, respectively. FIR digital filters are known by linear phase digital filters and it always possible to be designed with an exact linear phase response, so FIR filter transfer function has a linear phase is either symmetric or antisymmetric. Since the length of the impulse response can be either even or odd, the (FIR) digital filters are categorized to four types as:
Symmetric FIR and Odd M filter length, Asymmetric FIR and Odd M filter length, Symmetric FIR and Even M filter length

and Asymmetric and Even M filter length; all these types can be investigated and/or designed by four methods such as: Fourier transform method, window method, frequency sampling method design, and optimal design method. In IIR digital filters, the designing methods such as bilinear transformation method BLT is considered as most famous method, and it has been used for developing a procedure to design the most common and known digital (IIR digital filters) i.e. Butterworth, Chebyshev filters and etc.. There is other important designing methods are used, such as Impulse Invariant design method and pole-zero placement design method.

Since the design of fixed coefficient digital filters needs well-defined prescribed specifications. However, there are cases where the criteria are not available, or is time varying. The solution in these cases is to employ a digital filter with adaptive coefficients, known as adaptive filters. Since no specifications are available, the adaptive algorithm that determines the updating of the filter coefficients requires extra information that is usually given in the form of a signal. This signal is in general called a desired or reference signal, whose choice is normally a tricky task that depends on the application. Adaptive filters are considered nonlinear systems; therefore their behaviour analysis is more complicated than for fixed filters. The aim of digital filters is to isolate the signals which have been combined and to restore the signals which have in some way been distorted. And it's generally classified as Weiner and Kalman filters [14]. A derivation of Weiner filter using LMS algorithm in order to be used in this paper in Noise Cancellation Case Study.

A. *Adaptive Digital Filters-Overview and Applications*

An adaptive filter is a digital filter that has self-adjusting characteristics. It is capable of adjusting its filter coefficients automatically to adapt the input signal via an adaptive algorithm. Adaptive filters play an important role in modern digital signal processing (DSP) products in areas such as telephone echo cancellation, noise cancellation, equalization of communications channels, biomedical signal enhancement, active noise control, and adaptive control systems [1]. Adaptive filters work generally for adaptation of signal-changing environments, spectral overlap between noise and signal, and unknown, or time-varying, noise.

Generally, Adaptive filter is a computational device that attempts to model the relationship between two signals in real time in an iterative manner. Adaptive filters are often realized either as a set of program instructions running on an arithmetical processing device such as a microprocessor or DSP chip, or as a set of logic operations implemented in a field-programmable gate array (FPGA) or in a semicustom or custom VLSI integrated circuit [12]. However, ignoring any errors introduced by numerical precision effects in these implementations, the fundamental operation of an adaptive filter can be characterized independently of the specific physical realization that it takes. An adaptive filter is defined by four aspects:

1. Signals being processed by the filter
2. Structure that defines how the output signal of the filter is computed from its input signal
3. Parameters within this structure that can be iteratively changed to alter the filter's input-output relationship
4. adaptive algorithm that describes how the parameters are adjusted from one time instant to the next

A several different structures have been proven to be useful in practical applications based the input and desired signals and how these structures are related to the applications. System identification application: The desired signal is the output of the unknown system when excited by a broadband signal, such in the case of white-noise signal; Application of channel equalization: The channel equalization scheme consists of adding the originally transmitted signal skewed or corrupted by the channel plus ambient noise to an adaptive filter as the input signal, while the desired signal is a delayed version of the original signal. Prediction application: The desired signal is a forward or ultimately a backward version of the adaptive-filter input signal. The adaptive filter represents the model for the input signal after convergence and can be used as a prediction model for the input signal. Application of signal enhancement (Noise cancellation): The signal is corrupted by ambient noise, and a noise-related signal is available (measurable) used as an input to the adaptive filter with a noise-corrupted signal playing the function of the desired signal, the output error would be an improved version of the signal after convergence [13].

Noise is known to be a random process and adaptive filters are capable of changing their impulse response to filter out the input correlated signal.

B. *Adaptive Filtering Principle Statement*

Adaptive filters have the ability to monitor the signal under non-stationary conditions in an adaptive way. In order to alter the behaviour in time, it has the unique characteristic of self-modifying its frequency response and allowing the filter to adjust the response to the change in input signal characteristics. The General structure/configuration principle of an adaptive filter is shown Fig. 1. A sample from of a digital input signal $x(n)$ is fed into a device, called an adaptive filter that computes a corresponding output signal sample $y(n)$ at time$n$.

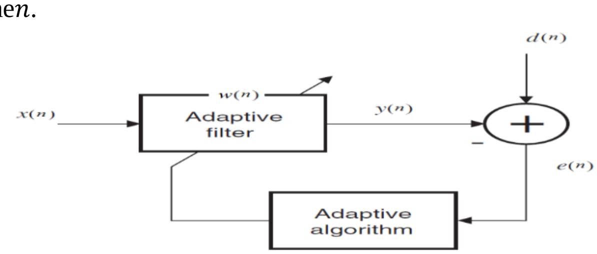

Figure 1. Adaptive filter Structure

The goal is the filter input signal, $x(n)$, with an adaptive filter in such a way that it matches the desired signal, $d(n)$. The structure of the adaptive filter is not important; except for the fact that it contains adjustable parameters whose values affect how $y(n)$ is computed. The output signal is compared to a desired signal $d(n)$, called the desired response signal or reference signal (that usually includes some noise component), by subtracting the two samples at time n. This difference signal, given by Eq. (1):

$$e(n) = d(n) - y(n) \tag{1}$$

The error signal $e(n)$ is fed into a procedure which alters or adapts the parameters of the filter from time n to time $(n+1)$ in a well-defined manner and used by the adaptation algorithm to update the adaptive filter coefficient vector $w(n)$ according to some performance criterion. Generally, the whole adaptation process aims at minimizing some metric of the error signal, forcing the adaptive filter output signal to approximate the reference signal in a statistical sense. Unlike the fixed filter design, the filter coefficients can be changed here, adjusted based on the setting in which the filter is worked, and can therefore detect any possible changes in this environment. According to this concept adaptive filters can be adapted to the context set by these signals. If the environment changes the filter into a new collection of variables, however, adjustments for new features occur. The most popular adaptive filter is the finite impulse response (FIR) filter with the tapped-delay line structure in which the weights are updated by the Least Mean Square (LMS) algorithm proposed by Widrow and Hoff in 1959 in their study of a pattern-recognition machine known as the adaptive linear element, commonly referred to as the Adaline.

The goal of this paper is to review the digital filters and adaptive digital filters, types, and classifications and design methods; current methods of noise cancellation to improve the quality of speech and audio signal and to provide an understanding of the suitability of different models developed. Prior to this the next segment provides a brief overview and algorithm derivation and steps of the least mean square LMS method of adaptive noise cancellation and its implementation. Finally, for further thought, a perception of upcoming.

III. NOISE CANCELLATION USING LMS FIR ADAPTIVE FILTER

Adaptive filter of finite impulse response FIR with the most popular algorithm known by least mean square LMS algorithm will be discussed here in this section.

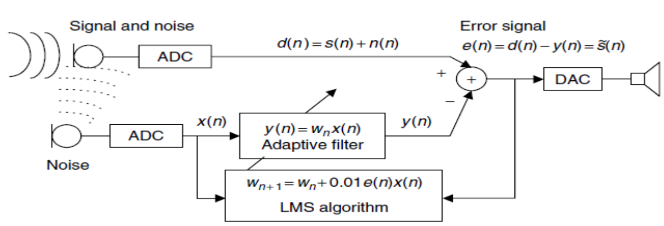

Figure 2. DSP System of noise cancellation adaptive filter [1]

A simple signal enhancement (noise canceller) application illustrated shows as block diagram in the Fig. 2. Here the system consists of two signal channel sources:

- Desired speech signal channel source: is to get the desired speech $s(n)$, this signal will be uncorrelated with the corrupting noise $n(n)$ signal due to the environment and the other noise channel source (noisy source or unwanted signal source), that is $d(n) = s(n) + n(n)$. Since the $s(n)$ signal is not correlated with the noise signal $n(n)$, so it possible to separated.
- Noise signal channel source: provides a noise signal $x(n)$ that will fed to the adaptive filter and it's already correlated to the corrupted noise $n(n)$.

"The adaptive filter contains a digital filter with adjustable coefficients for filtering each sample."

The adaptive filter then produces an estimate of noise $y(n)$, which will be subtracted from the corrupted signal $d(n) = s(n) + n(n)$ as $e(n) = d(n) - y(n)$.

When the noise estimate $y(n)$ equals or approximates the noise $n(n)$ in the corrupted signal $d(n)$, that $y(n) \approx n(n)$, the error signal $e(n) = s(n) + n(n) - y(n) \approx \check{s}(n)$ will approximate to clan speech signal $s(n)$. Hence, the noise is canceled.

A. Wiener Adaptive Filter Theory and LMS Algorithm

The Wiener filter [15] is a digital filter designed to minimize the mean square difference between the desired signal and the output of the filter. Occasionally, it is called a minimum mean square error filter. A Wiener filter [15] may be either a Finite Impulse Response (FIR) of linear equation formulation or an Infinite Impulse Response (IIR) of non-linear equation formulation.

The Wiener filter adjusts its weight(s) to produce filter output $y(n)$, which would be as close as the noise $n(n)$ contained in the corrupted signal $d(n)$. Hence, at the subtracted output, the noise is canceled, and the output $e(n)$ contains clean signal, Fig.3, shows Wiener Adaptive Filter for noise cancellation [1].

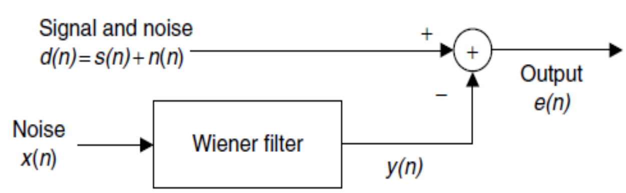

Figure. 3. Wiener Adaptive Filter for noise cancellation [1]

B. LMS Algorithm derivation

Considering a single-weight case of $y(n) = wx(n)$, and the error signal $e(n)$ is given by Eq. (1):

$$e(n) = d(n) - wx(n) \tag{2}$$

By solving the best weight w^*.Taking the square of the output error leads to

$$(e(n))^2 = (d(n) - wx(n))^2 = d^2(n) - 2d(n)wx(n) + w^2x^2(n) \tag{3}$$

Taking the statistical expectation of Eq. (3), we get:

$$E\big(e^2(n)\big) = E\big(d^2(n)\big) - 2wE(d(n)x(n)) + w^2E(x^2(n)) \tag{4}$$

Using the notations in statistics, we define:

$J = E\big(e^2(n)\big)$ ➔ MSE (mean squared error)
$\sigma^2 = E(d^2(n))$ ➔ Power of corrupted signal
$P = E(d(n)x(n))$ ➔ Cross-correl. between $d(n)$ and $x(n)$
$R = E(x^2(n))$ ➔ Autocorrelation

The statistical expectation as an average of the M signal terms, each being a product of two individual signal samples:

$$E\big(e^2(n)\big) = \frac{e^2(0)+e^2(1)+e^2(2)+\cdots\ldots\ldots e^2(M-1)}{M} = \frac{\sum_{n=0}^{M-1} e^2(n)}{M} \tag{5}$$

For a large sample number of M, Eq. (3) can be written as:

$$J = \sigma^2 - 2wP + w^2R \tag{6}$$

Since $\sigma^2, P\ and\ R$ are constants, J is a quadratic function of w.
The best (optimal) weight w^* value is when the MSE J is at the minmum.
To obtain w^*, taking a derivative of J with respect w and setting it to zero leads to:

$$\frac{dJ}{dw} = -2P + 2wR = 0 \quad =====> w^* = \ PR^{-1} \tag{7}$$

Since Wiener MSE Algorithm requires a lot of computations, including matrix inversion for a general multiple-taps FIR filter. By study the steepest descent algorithm as illustrated in Eq. (8) [1]:

$$w_{n+1} = w_n - \mu\frac{dJ}{dw} \tag{8}$$

Where is μ = constant controlling the speed of convergence.

$$J = (e(n))^2 = \big(d(n) - \ wx(n)\big)^2$$
$$\frac{dJ}{dw} = 2\big(d(n) - wx(n)\big)\frac{d\big(d(n)-wx(n)\big)}{dw} = -2e(n)x(n) \tag{9}$$

Substituting $\frac{dJ}{dw}$ into the steepest descent algorithm in Eq. (8), we achieve the LMS algorithm for updating a single-weight case as:

$$w_{n+1} = w_n + 2\mu e(n)x(n) \tag{10}$$

In general, with an adaptive FIR filter of length M, an extending the one-tap LMS algorithm without going through derivation, as shown in the following equations:

$$y(n) = w_n(0)x(n) + w_n(1)x(n-1) + w_n(2)x(n-2) + \ldots\ldots\ldots\ldots\ldots\ldots. + w_n(M-1)x(n-M+1) \tag{11}$$

$$\text{For} \quad 0 \le n \le M-1$$

$$w_{n+1}(i) = w_n(i) + 2\mu e(n)x(n-i) \tag{12}$$

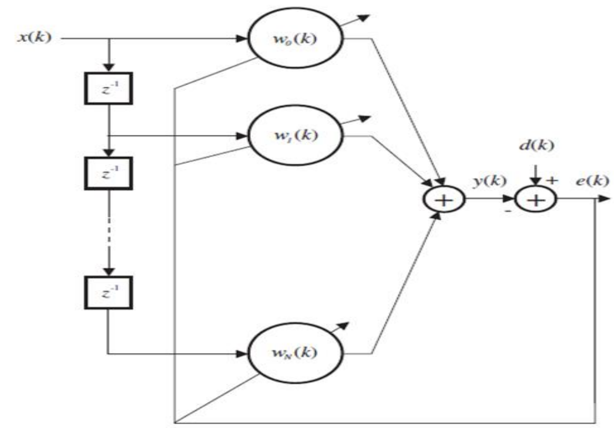

Figure.4. FIR LMS Wiener Adaptive Filter

The convergence factor μ is practically chosen to be as

$$\mu \approx \frac{3.9 * 10^{-10}}{M}$$

The FIR Filter length notated by M, and Filter order by N, where $N = M - 1$.

FIR Adaptive Filter using LMS Algorithm implementation steps:

1. Initialize $w(0), w(1), \ldots\ldots\ldots., w(N)$ to arbitrary values.
2. Read $d(n)$, $x(n)$, and performing digital filtering:
   $y(n) = w_n(0)x(n) + w_n(1)x(n-1) + \ \ldots + w_n(N)x(n-N)$
3. Compute the output error:
   $e(n) = d(n) - y(n)$
4. Update each filter coefficient using the LMS algorithm:
   $for\ i = 0\ to\ N\ do$:
   $w_{n+1}(i) = w_n(i) + 2\mu e(n)x(n-i)$

IV. CASE-STUDY: NOISE CANCELLATION IMPLEMENTATION

In this case-study of noise cancellation using adaptive filter a DSP system proposed consists of two ADC input sources Figure 5. The primary sensor with ADC captures the noisy speech, $d(n) = s(n) + n(n)$, which contains the clean speech $s(n)$ and noise $n(n)$ due to a noisy environment, while the Reference sensor with ADC resides where it picks up only the correlated noise and feeds the noise reference $x(n)$ to the adaptive filter.

The adaptive filter uses LMS algorithm to adjust its coefficients to produce the best estimate of noise $y(n) \approx n(n)$, which will be subtracted from the corrupted signal $d(n) = s(n) + n(n)$. The output of the error signal $e(n) = s(n) + n(n) - y(n) \approx \hat{s}(n)$ is expected to be a best estimate of the clean speech signal.

The corrupted noise $n(n)$ is highly correlated to the noise reference $x(n)$ since they are coming from the same noise source with an adequate distance between the primary and reference sensors, Fig.5.

The objective of the adaptive filter $W(z)$ is to compensate the amplitude and phase of the reference signal $x(n)$ in order to estimate the corrupted noise $n(n)$.

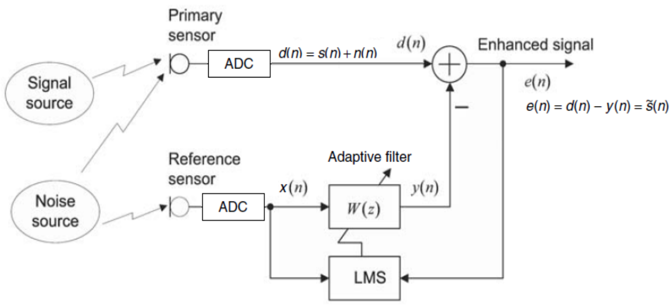

*Figure. 5. Adaptive noise cancellation using the **LMS** filter*

A. Implementation

The M-file script that implements the algorithm steps stated in the previous section using MATLAB in order to achieve adaptive filter for noise cancellation. It computes the filtered output, filters error and the filter weights for a given input and desired signal using the Least Mean Squares (LMS) algorithm.

B. LMS Algorithm Specification

- Speech corrupted by Gaussian noise with a power of 1 from the noise reference.
- Primary sensor 'Input Audio File .wav' $s(n)$: The signal of original audio speech as an input.
- 'Desired' $d(n)$: The signal from the original audio speech signal and the corrupted noise signal.
- Reference sensor 'Input' $x(n)$: The noise signal contains a Gaussian noise with a power of 1.
- 'err_sig' $e(n)$: The difference between the 'desired' and the filtered 'input'. It represents the estimated audio speech signal.
- Number of FIR filter taps = 40.
- Convergence factor μ for the ***LMS*** algorithm chosen to be $\mu = 0.01$

The LMS algorithm in the adaptive filter is trying to retrieve the original signal ('err_sig') from 1st Mic "Primary sensor" by filtering the 2nd Mic's signal "Reference sensor" and using it to cancel the noise in Primary sensor.

The coefficients/weights of the filter are updated (adapted) in real-time based on 'input' and 'err_sig'.

V. RESULTS AND DISCUSSION

In order to gain the prospective and anticipated performance by achieving the main objective of this paper, the simulation MATLAB file is tested and validated. We have suggested two input scenarios here: the first scenario is by allowing it to take a sinusoidal signal input, and the second is by giving it a real recorded Wav audio file to interpret. A random Gaussian white noise with a variance of 0.25 was suggested as the noise that distorted the original signal in both scenarios.

A. ' Primary sensor ' Input signal $s(n)$: The original signal is a sinusoidal input:

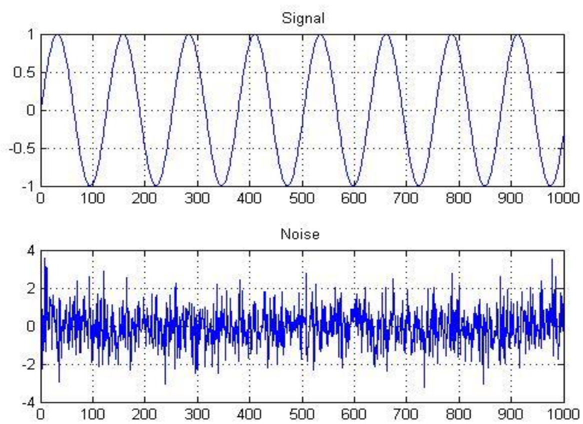

Figure.6. Original Sinusoidal signal and reference noise

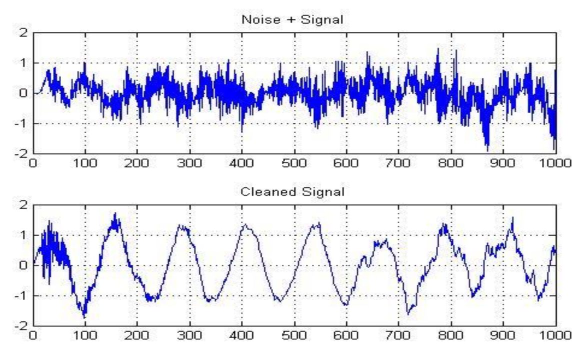

Figure. 7. Corrupted sinusoidal signal (signal + noise) and clean sinusoidal signal, convergence factor $\mu = 0.01$

In Fig. 6, displays the Sine wave input with the reference white noise that will damage the given input; while Fig.7, shows the original Sinewave input corrupted by noise and shows how it cleaned by applying the adaptive LMS algorithm. These are for the scenario of Sine wave input.

B. ' Primary sensor ' Input signal $s(n)$: The original signal is a real speech audio file .wav file:

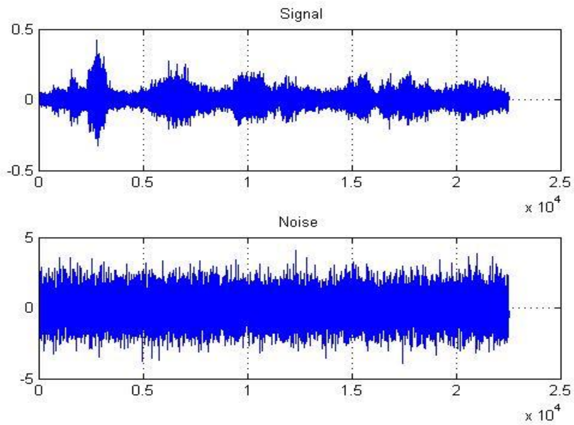

Figure 8. Original audio.wav signal and reference noise

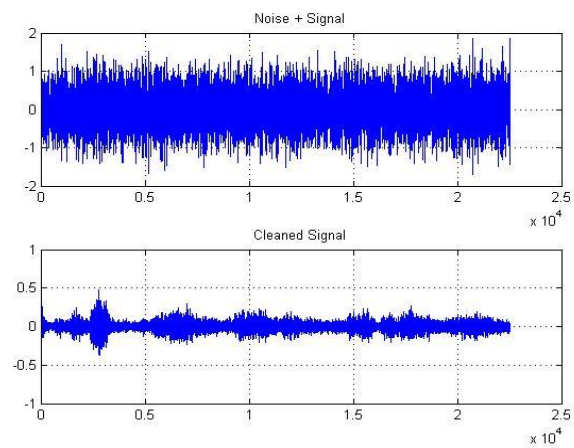

Figure 9. Corrupted Audio speeches signal (signal + noise) and clean audio signal, convergence factor $\mu = 0.01$

In the second scenario, Figures 8-9 shows audio speech waveforms and spectral plots for the original, reference nose, then corrupted speech and the cleaned speech using the proposed noise cancellation algorithm LMS.

It's clear that the enhanced speech waveform and spectrum are very close to the original corresponded input sources. In addition, the constant controlling the speed of convergence μ has been adjusted by giving different values and see the retained "cleaned" signals (try and review) to get an optimum value for this algorithm in turn to achieve an optimum noise cancelation.

The value of μ is critical. If μ is too large, the filter resolution is poor i.e. less efficient in noise cancelling. If μ is too small, the filter reacts slowly with more efficiency in noise cancelling. The selected value of μ is a compromise or trade-off issue.

Fig. 10, shows a comparison between 3 different values of μ and how it reflected directly to the output of the proposed algorithms in turn to get the best and must cleaned signal; we concluded that the best value of the convergence factor must be $\mu \leq 0.01$.

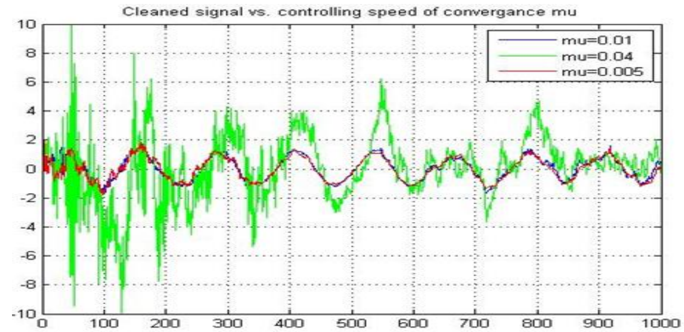

Figure 10. Comaprison between different outputs of adaptive filter using different convergance factors $\mu = 0.01\ 0.005$

VI. CONCLUTIONS

The main aim of the paper is to study, derive and implement an optimal adaptive filter that demonstrates good performance results in adaptive noise cancellation and produces the best approximation of the target signal from the noisy environment.

Adaptive filters are a very useful tool in signal processing as they have the capability to remove the noise from the signal even if the statistical data of the signal. The Least mean-square LMS algorithm is implemented, evaluated and compared against different noisy environment and different convergence factors.

To apply the adaptive noise cancellation effectively, the reference noise picked up by the reference sensor must be highly correlated with the noise components in the primary signal.

Noise cancellation using LMS algorithm for different types of noises is analysed using MATLAB tools and ready functions such `adaptfilt.lms` () which can be used for creating the desired filter.

It is observed that LMS algorithm is an effective technique for removal of noise as it has less complexity.

Many parameters can affect the efficiency of the adaptor filter such as the convergence factor μ and FIR Length M.

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